Stochastic Predetermination of Risk and Hedging Skills for Small Scale Entrepreneurs

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Authors’ contributions

This work was carried out in collaboration between both authors. Author IOL designed the study, performed the statistical analysis, wrote the protocol, managed the literature searches and wrote the first draft of the manuscript. Author EOA managed the analyses of the study. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2019/v33i630195

Abstract

Faced with the issue of hedging risk, small businesses entrepreneur are investing considerable resources in risk management systems in as much as to maximize profit and stay operational, as such, the types of risk are identified and quantified within each business. This paper focused on the application of stochastic processes to prove that risk could be predetermined and hence determine which kind of small business should be insured to mitigate money spent on insurance.

Keywords: Hedging; risk; stochastic processes; entrepreneurship.

1 Introduction

In nation’s economic development Small scale entrepreneurs plays a vital role and could engineer an economic growth but in a decade an estimated number of new small business survived their first two years...
of existence but only a few remain in operation past the five year mark. The reasons are trivial and is due to some regulatory burdens imposed on small business and multiple issues unaccounted for (i.e. operation, legal, financial and other risks facing small business entrepreneurs [1,2-4].

To this end, insurance is a major method that most people, businesses, and other organizations can use to transfer pure risks, by paying a premium to an insurance company in exchange for a payment of a possible large loss [5,6]. By using the law of large numbers, an insurance company can estimate fairly reliably the amount of loss for a given number of customers within a specific time. An insurance company can pay for losses because it pools and invests the premiums of many subscribers to pay the few who will have significant losses [7,8,9]. Most small-scaled entrepreneurs were involved in the payment of premium without estimating their risk to the premium paid and also observed if the risk involved is insurable.

According to Embrechts et al. [10], the theory of stochastic processes is the theory of the 20th century which first appeared through the applications of insurance and finance, so many researchers have written on these and they all believed that the financial markets are dominated by the laws of probability and the erratic behaviour of the stock market data were like the motion of small particles suspended in the fluid which is applicable to entrepreneurs. Several authors [11,12] established the sound theoretical basis in terms of risk-neutral valuation and equivalent martingale measures. Gerber [13] has introduced martingale method to risk theory, since then, papers have been investigating martingales in risk theory. Mainly these papers deal with assessing ruin probabilities. For a reveal of insurance-related use of martingale, see e.g. [14]. The martingale approach to premium calculation, which is considered here, has been pioneered by Delaen and Haezendonck [15].

In this paper, it was shown that common premium principles can be recovered by martingale methods. Another important paper in this context is by Sondermann [16], who considers arbitrage-free pricing for reinsurance. Hendrick [17] worked on the evaluation of value risk models, and recognition of these models by the financial and regulatory communities is evidence of their growing use. For example, the Besel committee on banking supervision endorsed the use of such a model in 1996 [18,19]. Having spelt out the risk management guidelines for derivatives in 1994, the same year in which the Bank for international settlement Fisher report urged financial intermediaries to disclose measures of value-at-risk publicly which led to the introduction of the Risk metrics database compiled by [20], for the use with third-party value-at-risk software by financial and non-financial firms. Notably in 2001, Ernest Eberbsin, Jenkallen and Jorn Kristen worked on the paper ‘Risk management based on stochastic volatility’ which became the founding ideas for [21], in their Master thesis, to jointly work on value at Risk using Stochastic Volatility models thereby giving an extension to Ebetiem; Kallen and Kristein work by investigating other stochastic Volatility model [22]. Risk measurement and risk pricing are also worked on by some researchers like [23].

1.1 Background of the study

All the stochastic processes used in this paper are defined on the probability space \((\Omega, \mathcal{F}, P)\)

Definition (Lévy Process)

An R-valued stochastic process \(\{X_t; t \geq 0\}\) is Lévy process if the following conditions are satisfied:

- For any choice of \(n \geq 1\) and \(0 \leq t_0 \leq t_1 \leq \cdots \leq t_n\) random variable \(X_{t_0}, X_{t_1} - X_{t_0}, X_{t_2} - X_{t_1}, \cdots, X_{t_n} - X_{t_{n-1}}\) independent (i.e. the process has independent increment)
- The distribution of \(X_{t+s} - X_t\) does not depend on \(s, i\)
- For all \(\epsilon > 0\) it holds that
  \[\lim_{t \to s} P(|X_s - X_t| > \epsilon) = 0\]
  i.e. stochastic continuity
• $X_t$ is right continuous for $t \geq 0$ and has left limit for $t > 0$ satisfying the stationary and independent increment property together with a mild sample-path regularity condition are also referred to a Lévy process.

This plays a good role in insurance.

**Definition 1.1:**

Standard Brownian motion $W = (W_t) \geq 0$ real value stochastic process which satisfied the following condition

(a) $W$ start at zero: $W_0 = 0$ a.s.  
(b) $W$ has independent increment for any partition $0 \leq t_0 \leq t_1 \leq \cdots \leq t_n < \infty$ and any $k$, the random variable (rvs) $W_{t_1} - W_{t_0}, W_{t_2} - W_{t_1}, \ldots, W_{t_k} - W_{t_{k-1}}$ are independent.
(c) $W$ has Gaussian increment for any $t > 0$, $W_t$ is normally distributed within 0 and variance $t$, i.e. $W_t \sim \mathcal{N}(0, t)$ .
(d) $W$ has a.s. continuous sample paths.

The condition (b) and (c) are referred to as $W$ has stationary and independent increment moreover, the increments are normally distributed.

**Theorem 1.1: (Existence of Brownian Motion)**

Let $(\Omega, \mathcal{F}, P)$ be a probability space on which countably many $\mathcal{N}(0, 1)$ independent random variables are defined. Then there exists a 1-dimensional Brownian motion on $(\Omega, \mathcal{F}, P)$.

**Remark:**

From the last Theorem, we have the existence of a d-dimensional Brownian motion on $(\Omega, \mathcal{F}, P)$ since such a stochastic process merely a d-tuple of independent, 1-dimensional Brownian motion.

Indeed, we see from the foregoing definition of a d-dimensional Brownian motion that with

$$W(t) = (W_1(t), W_2(t), W_3(t) , \ldots, W_d(t)) , \text{ } t \in (0, \infty)$$

we have

(i) $E(W(t)) = 0$ and $E(W_j(t))^2 = t \quad \forall \ t \geq 0, j = 1, 2, \ldots, d.$  
(ii) $E\left(W_j(t)W_k(s)\right) = \delta_{j,k} t \wedge s = \min\{t, s\} \quad s, t \geq 0, j = 1, 2, \ldots, d$ since

$$E\left(W_j(t)W_k(s)\right) = E\left(\left(W_j(s) + W_j(t) - W_j(s)\right)W_k(s)\right)$$

$$= E(W_j(s)W_k(s) + E(W_j(t) - W_j(s))W_k(s))$$

$$= E(W_j(s)W_k(s))\delta_{j,k}s \quad \text{for } s \leq t .$$

**Remark (Sample Path)**

(i) Let $W = (W_1, \ldots, W_d)$ be the d-dimensional Brownian motion on $(\Omega, \mathcal{F}, P)$

Then for each $\omega \in \Omega$, the map

$$t \rightarrow W(t, w) = W_1(t, w), W_2(t, w), \ldots, W_d(t, w)$$

is called a sample path of $W$. 

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Proposition 2.1.4:

Suppose W defined as above. The P- a.s., the sample paths of W are nowhere differentiable.

A rather unpleasant consequence of this result is that W has unbounded variation on each interval I says

$$\sup_{\Delta} \sum_{i=1}^{n} |W_{t_i}(w) - W_{t_{i-1}}(w)| = \infty$$

(2)

for P-almost all w ∈ Ω, Delta being a possible partitions

$$\Delta = \{t_0, \ldots, t_n\}$$

and ‘sup’ taken over all such partition.

Since standard integration theory for functions with bounded variation does not work.

i.e. the symbol

$$\int_0^t Y_s(w) dW_s(w)$$

For some stochastic process (Y) has no immediate meaning at first but with Ito calculus.

To solve the equation above, we consider \(L^2_{\mu}(\mathbb{R}_+, dt)\) of all function.

$$f: \mathbb{R}_+ \rightarrow L^2(\Omega, F, P)$$

such that the stochastic process \(t \rightarrow f(t, w(t))\) \(t \in \mathbb{R}_+\) is adapted and lies in \(L^2_{\mu}(\mathbb{R}_+, dt)\) for \(X \in L^2_{\mu}(\mathbb{R}_+, dt)\).

Then for \(x_0\) a fixed random variable on \((\Omega, F, P)\) and \(f, g \in L^2_{\mu}(\mathbb{R}_+, \Omega)\), the stochastic integral expression

$$x_0 + \int_{t_0}^t g(s, X(s)) ds + \int_{t_0}^t f(s, X(s)) dW(s), t \in \mathbb{R}_+, X \in L^2_{\mu}(\mathbb{R}_+, dt)$$

(3)

where the first integral is Riemann and the second is an Ito Integral.

Theorem (W has a finite quadratic variation)

Suppose W as above, but change (c) the normality assumption to (c) for all t, \(W_t \sim N(0, \sigma^2 t, \sigma > 0)\) then for \(n \rightarrow \infty\)

$$\sum_{i=1}^{n} |W_{t_i} - W_{t_{i-1}}|^2 \overset{L^2}{\rightarrow} \sigma^2 t$$

where \(\{t_0, \ldots, t_{i-1}\}\) such that

$$\sup_{t} |t_i - t_{i-1}| \rightarrow 0$$

(4)

where \(\overset{L^2}{\rightarrow}\) denote convergence in \(L^2(\Omega, F, P)\) Under a slight extraction in the partition used, \(L^2\) – convergence can be replaced by by almost sure convergence.
Definition: (Poisson)

The stochastic counting process \( N = N(t) \) homogeneous Poisson process with rate (intensity) \( \lambda > 0 \)

(a) \( N(0) = 0 \)

(b) \( N \) has stationary, independent increments.

A (c) For all \( 0 \leq s < t < \infty \), \( N(t) - N(s) \sim \text{Pois}(\lambda(t - s)) \)

\[
P(N(t) - N(s) = k) = e^{-\lambda(t-s)} \frac{(\lambda(t-s))^k}{k!}, \quad k \in \mathbb{N}
\]

Both processes are Lévy processes, the key difference lies in behaviour of their sample path. Brownian motion has continuous sample paths, whereas the Poisson process is as a counting process, a jump process.

The values of product and asset in the market do behaviour like Brownian motion and sometimes do the jump. There is behavioural tread of risk in any business.

## 2 Insurance and Ruin Process

Risk management is the identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and or impact of unfortunate events or to maximize the realization of opportunities. Risks can come from uncertainty in financial markets, threats from project failures (at any phase in design, development, production, or sustainment life-cycles), legal liabilities, credit risk, accidents, natural causes and disasters as well as deliberate attack from an adversary, or events of uncertain or unpredictable root-cause [24-26]. Several risk management standards have been developed including the Project Management Institute, the National Institute of Standards and Technology, Actuarial societies, and ISO standards. Methods, definitions and goals vary widely according to whether the risk management method is in the context of project management, security, engineering, industrial processes, financial portfolios, actuarial assessments, or public health and safety.

The strategies to manage threats (uncertainties with negative consequences) typically include transferring the threat to another party, avoiding the threat, reducing the negative effect or probability of the threat, or even accepting some or all of the potential or actual consequences of a particular threat, and the opposites for opportunities (uncertain future states with benefits).

Insurance is the equitable transfer of the risk of a loss, from one entity to another in exchange for payment. It is a form of risk management primarily used to hedge against the risk of a contingent, uncertain loss. An insurer, or insurance carrier, is a company selling the insurance; the insured, or policyholder, is the person or entity buying the insurance policy. The amount of money to be charged for a certain amount of insurance coverage is called the premium. Risk management, the practice of appraising and controlling risk, has evolved as a discrete field of study and practice. The transaction involves the insured assuming a guaranteed and known relatively small loss in the form of payment to the insurer in exchange for the insurer's promise to compensate (indemnify) the insured in the case of a financial (personal) loss. The insured receives a contract, called the insurance policy, which details the conditions and circumstances under which the insured will be financially compensated.

Risk-sharing between buyer and seller, seller has to bear a part of the remaining risk, participants in derivative markets are faced with a large amount of credit risk and it is believed that all this risk can be hedged away [27].

In Insurance application \( N(t) \) stands for the number of claims in time interval \((0,t]\) in a well defined portfolio.
If the arrival of the $n$th claim is denoted by $S_n$, then

$$N(t) = \sup\{n \geq 1, S_n \leq t\} , \ t \geq 0$$

The inter-arrival times $T_1, T_k = S_k - S_{k-1}, \ k = 2, 3, \ldots$ are independent, identically exponentially distributed ($\exp(\lambda)$) with finite mean $\mathbb{E}[T] = \frac{1}{\lambda}$. The latter property also characterises the homogeneous Poisson process. The claim size $(X_k)_{k \in \mathbb{N}}$ is at first assumed to be the distribution function $F(F(0) = 0)$ and finite mean $\mu = \mathbb{E}X_1$. The $X_k$ denotes the claim size occurring at time $S_k$.

As a consequence of the above, the total claim amount up to time $t$ is given by

$$S(t) = \sum_{k=1}^{N(t)} X_k$$

The latter random variable is compound poisson random variable

$$G_t(x) = \mathbb{P}[S(t) \leq x] = \sum_{n=0}^{\infty} \mathbb{P}_n(t) F^n(x)$$

$$= \sum_{n=0}^{\infty} e^{-\lambda t} \frac{\lambda^t}{n!} F^n(x), \ x, t \geq 0$$

$F(x)$ = Distribution of single claim

$\mathbb{P}_n(x)$ = Distribution of number of claims

$G_t(x)$ = Distribution of total claim

Its precise calculation and indeed statistical estimation in practice from a key area of research in Insurance risk theory.

To equalize the loss caused by claims and to eventually make profit; an insurance company impose a premium on its clients, $k(t)$ be the total premium $k(t) = ct$ income of insurance company in $[0, t]$ in case $S(t) < k(t)$, the company has made a profit of

$$k(t) - S(t)$$

besides the liability process $(S(t))_{t \geq 0}$ an insurance company cashes premiums to compensate the losses. The premium process $(\mathbb{P}(t))_t$ is assumed to be linear (deterministic), i.e.

$$u + ct$$

where $u \geq 0$ stands for initial capital and $c > 0$ is the constant premium rate chosen in such a way that the company (or portfolio) has fair chance of “survival”. The following random variable is crucial in this context: denote by $\tau$ the ruin of the risk process.

$$U(t) = u + ct - S(t), \ t \geq 0 \quad (\text{net – profit})$$

i.e

$$\tau = \inf\{t \geq 0: \ U(t) < 0\}$$

The associated ruin probabilities are defined as

$$\Psi(u, T) = \mathbb{P}(\tau \leq T), \ T \leq \infty$$

The ruin probability decreasing with increasing initial capital $u$. 

i.e
For $\Psi(u, \infty)$, the infinite horizon ruin probability. We write $\Psi(u)$. The so-called net-profit condition is
\[ c - \lambda u > 0 \quad (\text{Cramer Lundberg risk process}) \]

\[ \lim_{u \to \infty} \Psi(u) = 0 \] within the condition above is always assumed. It says that on the average, we obtain a higher premium income than a claim loss.

The basic risk process can now be rewritten as
\[ U(t) = u + (1 + \sigma)\lambda t - S(t) \]
where $\lambda t = \mathbb{E}S(t)$ and $\sigma = \theta / (\lambda t) - 1 > 0$ is the so-called safety loading which guaranteed “survival”.

Definition the stochastic process $(U(t))_t$ defined in [28] with net-profit condition [29] is called the Cramer-Lundberg risk process.

Theorem: According to Embrechts et al. [10], the Cramer-Lundberg model as above is explained as follows:
\[ 1 - \Psi(u) = (1 - \rho)\sum_{n=0}^{\infty} \rho^n F_1^n(u), \quad u \geq 0 \]

Where $\rho = \frac{\lambda u}{c} < 1$ and the integrated tail $F_1$ is defined as
\[ F_1 = \int_0^x \mathbb{F}(y) \, dy, \quad x \geq 0 \]

### 3 Martingale Theory

Definition:

A stochastic process $M = (M_t)_{t \in \mathbb{T}}$ in the filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ is an $\mathbb{F}$ martingale (sub – martingale, super- martingale, respectively) if

a) $M$ is $\mathbb{F}$ adapted, integrable and

b) for all $0 \leq s \leq t$, $\mathbb{E}(M_t | \mathcal{F}_s) = (M_s)$

\[ \text{i.e.} \]

Submartingale
\[ \mathbb{E}(M_t | \mathcal{F}_s) \geq (M_s) \quad \text{for all} \ 0 \leq s \leq t \]

Supermartingale
\[ \mathbb{E}(M_t | \mathcal{F}_s) \leq (M_s) \quad \text{for all} \ 0 \leq s \leq t \]

Martingale
\[ \mathbb{E}(M_t | \mathcal{F}_s) = (M_s) \quad \text{for all} \ 0 \leq s \leq t \]

We simply say $M$ is a martingale (submartingale, supermartingale) concerning the natural filtration.

Theorem (Martingale stopping theorem)

Let $M$ be an $\mathbb{F}$ – martingale (submartingale, supermartingale) and $\mathcal{T}$ an $\mathbb{F}$ – stopping then, assume that $\mathbb{F}$ sightly continuous; then also stopped stochastic process
\[ M_{\mathcal{T} \wedge t}, \quad t \geq 0 \]
is an $\mathbb{F}$ – martingale (submartingale, supermartingale)

Moreover for all $t \geq 0$

$$\mathbb{E} (M_{\tau_A t} | \mathcal{F}_t) = \langle \ge \le \rangle M_{\tau_A t}$$

The next theorem yields a precise formulation statement that “all martingale converge”.

Theorem (Martingale convergence theory)

Let $M$ be an $\mathbb{F}$ – supermartingale such that

$$\sup_{t \geq 0} \mathbb{E} M_t < \infty$$

if $\mathbb{F}$ sight continuous ,then $M_{\tau} := \lim_{\tau \to \tau} M_{\tau}$ exists $\mathbb{P}$ – a.s, moreover

$$\mathbb{E}[M_{\tau}] < \infty$$

An immediate consequence of the above is that positive (or indeed negative) martingales converge almost surely. A third important category of results is the so-called martingales inequality. For our purposes, the following martingales related to Brownian motion and the homogeneous Poisson process is important.

**Proposition:**

(a) Suppose $N$ is a homogeneous Poisson process with intensity $\lambda > 0$, then $(N(t) - \lambda(t))_t$ is a martingale.

(b) Considering the Cramer- Lundberg model, let

$$\theta(r) = \lambda(\mathbb{E}e^{rX_1} - 1) - cr$$

For those $r$ -values for which $\mathbb{E}e^{rX_1}$ exists, then

$$(M_r(t))_t := \langle \exp(-rU(t) - \theta(r)t) \rangle_t$$

**Proof:**

With stopping theorem above, thus result yields important information on the probabilities of ruin $\Psi(u, T)$.

The proof of (13) is fairly easy since $(U(t))_t$ is a (strong) Markov process for $0 \leq s \leq t$,

$$\mathbb{E}(M_r(t) | \mathcal{F}_s) = \mathbb{E}(\exp(-rU(t) - \theta(r)t) | \mathcal{F}_s)$$

$$= \mathbb{E} \left[ \exp(-rU(t) - \theta(r)t) | \mathcal{F}_s \right]$$

$$= \mathbb{E} \left[ e^{-rU(t)} e^{-\theta(r)t} | \mathcal{F}_s \right]$$

$$= \mathbb{E} \left[ (e^{-rU(t) - U(s) + U(s)}) | \mathcal{F}_s \right] e^{-\theta(r)t}$$

$$= \mathbb{E} \left[ e^{-r(U(t) - U(s)) | \mathcal{F}_s} e^{-\theta(r)t} \right]$$

$$= \mathbb{E} \left[ e^{-r(U(t) - U(s)) | \mathcal{F}_s} \exp(-rU(s) - \theta(r)t) \right]$$

$$= \mathbb{E} \left[ \exp(-r(U(t) - U(s)) | \mathcal{F}_s) \exp(-rU(s) - \theta(r)t) \right]$$

From equation (7) and (5) being substituted

$$= \mathbb{E} \left( \exp \left( r \sum_{i=N(s)+1}^{N(s)} X_i \right) | \mathcal{F}_s \right) \exp(-rU(s) - \theta(r)t)$$
from (13)

\[= \exp(\lambda(\mathbb{E}e^{rX_s} - 1)(t - s))\exp(-rU(s) - \lambda(\mathbb{E}e^{rX_s} - 1)t + crs)\]
\[= \exp(-rU(s) - \theta(r)s)\]
\[= M_s(t)\]

**Proposition:**

Suppose \(W = (W_t)_t\) is standard Brownian motion, then

(a) \(W\) and \((W_t^2 - t)\) are martingales

(b) For any \(\mu \in \mathbb{R}, \sigma > 0\), denote \(W_{\mu, \sigma}(t) = \mu t + \sigma W_t\), then \((W_{\mu, \sigma}(t))_t\) is called Brownian motion with drift \(\mu\) and variance \(\sigma^2\). For each \(\beta \in \mathbb{R}\) the following process

\[\left( \exp\{\beta W_{\mu, \sigma}(t) - (\mu \beta + \sigma^2 \beta^2/2)t\} \right)_t\]

is a martingale associated to Brownian motion, called exponential martingale.

**Proof:**

(a) If \(M(t) = (W_t^2 - t)_t\) for \(0 \leq s \leq t\)

\[\mathbb{E}(M_t | \mathcal{F}_s) = \mathbb{E}[W_t^2 - t | \mathcal{F}_s] = \mathbb{E}(W_t^2 | \mathcal{F}_s) - t\]

\[\mathbb{E}[(W_t - W_s)^2 | \mathcal{F}_s] - t = \mathbb{E}[(W_t - W_s)^2 | \mathcal{F}_s] - 2\mathbb{E}[W_t(W_t - W_s) | \mathcal{F}_s] + \mathbb{E}(W_t^2 | \mathcal{F}_s) - t\]

\[\Rightarrow t - s - 0 + W_s^2 - t = W_s^2 - s\]

is martingale.

(b) Denote \(W_{\mu, \sigma}(t) = \mu t + \sigma W_t\) to show that

\[\left( \exp\{\beta W_{\mu, \sigma}(t) - (\mu \beta + \sigma^2 \beta^2/2)t\} \right)_t\]

Is martingale, thus

\[\mathbb{E}(\exp\{\beta W_{\mu, \sigma}(t) - (\mu \beta + \sigma^2 \beta^2/2)t\} | \mathcal{F}_s)\]

\[= \mathbb{E}[\exp\{\beta W_{\mu, \sigma}(t)\} | \mathcal{F}_s] \exp\{- (\mu \beta + \sigma^2 \beta^2/2)t\}\]

\[= \mathbb{E}[\exp\{\beta(W_{\mu, \sigma}(t) - W_{\mu, \sigma}(s)) + W_{\mu, \sigma}(s)) | \mathcal{F}_s] \exp\{- (\mu \beta + \sigma^2 \beta^2/2)t\}\]

\[= \mathbb{E}[\exp\{\beta(W_{\mu, \sigma}(t) - W_{\mu, \sigma}(s)) | \mathcal{F}_s] [\exp\{\beta W_{\mu, \sigma}(s))\} \times [\exp\{- (\mu \beta + \sigma^2 \beta^2/2)t\}]\]

Recall \(W_{\mu, \sigma}(t) = \mu t + \sigma W_t\)

\[= \mathbb{E}[\exp\beta(\mu t + \sigma W_t - \mu s - \sigma W_s) | \mathcal{F}_s] [\exp\{\beta W_{\mu, \sigma}(s))\} \times [\exp\{- (\mu \beta + \sigma^2 \beta^2/2)t\}]\]

\[= \mathbb{E}[\exp\beta(\sigma W_t - \sigma W_s) | \mathcal{F}_s] [\exp\{\beta \mu(t - s))\} [\exp\{\beta W_{\mu, \sigma}(s))\} \times [\exp\{- (\mu \beta + \sigma^2 \beta^2/2)t\}]\]

\[= \exp\left[\frac{\beta^2 \sigma^2(t - s)}{2}\right] [\exp\beta \mu(t - s))\} [\exp\{\beta W_{\mu, \sigma}(s))\} \times [\exp\{- (\mu \beta + \sigma^2 \beta^2/2)t\}]\]

\[= \exp\left[\beta W_{\mu, \sigma}(s) - (\mu \beta + \sigma^2 \beta^2/2)s\right]\]

as such is martingale.

Since risks structural movement is like that of Brownian motion which is martingale and converges within a range, hence could be predetermine and managed.
4 Mitigating Risks for Entrepreneurs

There are many logical ways of reducing risks, such as:

4.1 Avoidance

Avoidance is the elimination of risk. You can avoid the risk of a loss in the stock market by not buying or shorting stocks; example: to stay away from the risk of divorce, is by not marrying; Many Entrepreneurs avoid legal risk by not going into an illegal business. Of course, not all risks can be avoided. Notable in this category is the risk of death. But even where it can be avoided, it is often not desirable. By avoiding risk, you may be avoiding many pleasures of life, or the potential profits that result from taking risks. Those who minimize risks by avoiding activities are usually bored with their life and don't make much money. Virtually any activity involves some risk. Where avoidance might not be possible or desirable, but necessary in some cases.

4.2 Loss control

Loss control can either be effected through loss prevention, which is reducing the probability of risk, or loss reduction, which minimizes the loss.

Loss prevention requires identifying the factors that increase the likelihood of a loss, then either eliminating the factors or minimizing their effect. For instance, speeding and driving drunk greatly increase auto accidents. Not driving after drinking alcohol is a method of loss prevention that reduces the probability of an accident. Driving slower is an example of both loss prevention and loss reduction, since it both reduces the probability of an accident and if an accident does occur, it reduces the magnitude of the losses, since accidents at slower speeds generally cause less damage. Most businesses actively control losses because it is a cost-effective way to prevent losses from accidents and damage to property, and generally becomes more effective the longer the business has been operating, since it can learn from its mistakes.

4.3 Risk retention

Risk retention is handling the unavoidable or unavowed risk internally, either because insurance cannot be purchased or it is too expensive for the risk, or because it is much more cost-effective to handle the risk internally. Usually, retained risks occur with greater frequency, but have a lower severity. An insurance deductible is a common example of risk retention to save money since a deductible is a limited risk that can save money on insurance premiums for larger risks. Businesses actively retain many risks — what is commonly called self-insurance — because of the cost or unavailability of commercial insurance.

4.4 Passive risk retention

Passive risk retention is retaining risk because the risk is unknown or because the risk-taker either does not know the risk or considers it a lesser risk than it is. For instance, smoking cigarettes can be considered a form of passive risk retention, since many people smoke without knowing the many risks of disease, and, of the risks they do know, they don't think it will happen to them. Another example is speeding. Many people think they can handle speed, and that, therefore, there is no risk. However, there is always greater risk to speeding, since it always takes longer to stop or change direction, and, in a collision, higher speeds will always result in more damage or risk of serious injury or death, because higher speeds have greater kinetic energy that will be transferred in a collision as damage or injury. Since no driver can foresee every possible event, there will be events that will happen that will be much easier to handle at slower speeds than at higher speeds. For instance, if someone fails to stop at an intersection just as you are driving through, then, at slower speeds, there is a greater chance of avoiding a collision, or, if there is a collision, there will be less
damage or injury than would result from a higher speed collision. Hence, speeding is a form of passive risk retention.

4.5 Noninsurance transfers of risk

The 3 major forms of noninsurance risk transfer are by contract, hedging, and, for business risks, by incorporating. A common way to transfer risk by contract is by purchasing the warranty extension that many retailers sell for the items that they sell. The warranty itself transfers the risk of manufacturing defects from the buyer to the manufacturer. Transfers of risk through contract is often accomplished or prevented by the contract, which may limit liability for the party to which the clause applies.

4.6 Derivative securities

Derivative securities is a financial contract whose value at an expiration date T, which is written into the contract is determined by the price process of some financial asset or instruments called underlying financial assets, up to time T. such as Option, swaps, forwards and futures.

5 Conclusion

Entrepreneurs are exposed to risk in an unstable market which is Brownian in nature and could be predetermined and hedged for un-hedgeable risks: Risk-management might prove helpful in dealing with these through insurance. When the risk and capital u are high so as not to fall into ruin $\Psi$ due to the premium and fees paid to insurance companies as identified in [28,30,13,29]. Therefore, a small scale entrepreneur could consider the risk and price involved in the business before insuring if necessary, otherwise, look for a befitting way of hedging the risk in his/her business.

6 Recommendation

The following recommendations were considered appropriately for entrepreneurs:

- They should plan, develop and research their market for risk fact.
- They should contact a risk management specialist to study, evaluate financially, determine the legal structure and give the best feedback on risk mitigating skill for the business.
- Insurance should be the last result of mitigating risk for a small-scaled entrepreneur.

Competing Interests

Authors have declared that no competing interests exist.

References


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Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sdiarticle4.com/review-history/51561